Économie publique/Public economics
03-04 | 1999/1-2
La politique familiale

Adult-Equivalence Scales : A Brief Survey

Charles Blackorby

Édition électronique
URL : http://journals.openedition.org/economiepublique/1952
ISSN : 1778-7440

Éditeur
IDEP - Institut d'économie publique

Édition imprimée
Date de publication : 15 juillet 1999
ISBN : 2-8041-3059-2
ISSN : 1373-8496

Référence électronique
Charles Blackorby, « Adult-Equivalence Scales : A Brief Survey », *Économie publique/Public economics* [En ligne], 03-04 | 1999/1-2, mis en ligne le 12 février 2007, consulté le 22 avril 2019. URL : http://journals.openedition.org/economiepublique/1952

économiepublique

public economics

Revue de **l'Institut d'Économie Publique**

Deux numéros par an

nº 3-4 – 1999/1-

2

Editions De Boeck Université
Rue des Minimes 39, B-1000 Bruxelles

Imprimé en Belgique

ISSN 1373-8496
Dépôt légal 1999/0074/219
ISBN 2-8041-3059-2

économiepublique sur internet : www.economie-publique.fr

Centre de la Vieille-Charité

2, rue de la Charité – F-13002 Marseille

La revue économiepublique bénéficie du soutien du Conseil régional Provence-Alpes-Côte d'Azur

ISSN 1373-8496

Adult-Equivalence Scales : A Brief Survey[1]

Charles BLACKORBY
Economics Department,
The University of British Columbia
et GREQAM, Marseille

1. Introduction

Equivalence scales are used explicitly or implicitly in many policy environments. In general these are policies that must apply to households of different size and composition. That is, the number of adults, the number of children, their ages, their health status, and their geographic locations are different but one general policy must cover them all. Even in the simplest economic environment this involves a host of problems that entail recourse to considerations methodological, theoretic, and econometric. There is no simple solution to these problems ; in fact, there are many solutions and it is not possible to make this choice on simple theoretical or empirical grounds. However, understanding the underlying theoretical and econometric issues is essential if one is to make a reasoned and wise choice.

There are two fundamental problems that must be resolved. The first is that one is trying to deduce information about the well-being of individuals from observable data. Even in the case of a single individual this requires a host of well-known assumptions.[2] However, for the most part, the data to which we have access is generated by households and not individuals, whereas, it is the well-being of the individuals in which we are interested. The second problem arises from the fact that we are trying to compare the well-being of the individuals in different households. This requires that we can at least compare the levels of utilities of different individuals, an assumption that is at odds with standard neo-classical claims. In fact, as we shall see, every class of equivalence scales requires

1. Most of this material is drawn from my joint work with David Donaldson [1987, 1988, 1991, 1993a, 1993b, 1994].

2. The strong axiom of revealed preference guarantees that the data is generated by a well-behaved preference ordering ; one must in addition assume that this preference ordering in fact yields information about the well-being of the individual.

additional assumptions about the comparability of utilities over and above the level comparability assumption required to even broach the subject. Equivalence scales can resolve these problems although, as mentioned above, there is no unique way to do this.

If, for example, a household of two adults and two children with an income of $ 30,000 has an equivalence value of three, then the household members can be regarded as enjoying the same level of well-being as a 'reference' single adult with $ 10,000, facing the same prices the household does. For social-evaluation purposes the household is equivalent to four reference single adults with incomes of $ 10,000 each. Thus, adult-equivalence scales permit the conversion of a complex problem in social evaluation into one where each household consists of a single person, and all households have the same preferences. Concealed in the definition of the equivalence scale are assumptions about how the household allocates its income for both private and public goods as well as other types of joint consumption. In addition, by its very definition, the equivalence scale entails interpersonal comparison of utility levels.

To make this argument as transparent as possible, I present in detail a class of relative equivalence scales making clear all of the required assumptions and their implications. Then, I make a modest change in one assumption and demonstrate that the implications are not only surprisingly strong but note also that they are unlikely to be unacceptable on a variety of grounds. I conclude with some brief remarks about the literature.

2. The Model

Consider a simple model of utilities and preferences of household members in an economic environment with m private goods. The population consists of n people, $n \geq 2$, and it is grouped into H households, $2 \leq H \leq n$. Households are indexed by h and are described by vectors of characteristics. $\mathbf{A}$ is the set of all possible vectors of characteristics ($|\mathbf{A}| \geq 2$) and α_h describes household h. $N(\alpha_h)$ is the number of people in household h, and we therefore have

$$\sum_{h=1}^{H} N(\alpha_h) = n. \tag{2.1}$$

α may contain the names of household members, their ages, sexes, locations, states of health, and so on.

Households are assumed to be income-sharing groups of people which make joint consumption decisions. Households, therefore, have preferences that rationalize their demand functions while individuals experience utilities. Following Samuelson [1956], each household is assumed to maximize a continuous, increasing, quasi-concave social-evaluation function of its members' utilities,[3] and individual utility levels, in turn, depend on individual consumption of private, public and semi-public household goods.

The household social-evaluation function may be normalized to measure the equally-distributed-equivalent utility of the household's members : it is the utility level which, if enjoyed by all, would be equivalent, according to the household's social-evaluation function, to the utility vector actually experienced. If a household with characteristics $\alpha \in A$ contains l members with utilities $(u^1, \ldots, u^l)$, then the equally-distributed-equivalent utility function $\widetilde{W}(\cdot, \alpha)$ corresponding to the household social-evaluation function $W(\cdot, \alpha)$ is given by

$$\widetilde{W}(u^1, \ldots, u^l, \alpha) = \tilde{u}, \tag{2.2}$$

where $\tilde{u}$ is defined by

$$W(\tilde{u}, \ldots, \tilde{u}, \alpha) = W(u^1, \ldots, u^l, \alpha). \tag{2.3}$$

$\widetilde{W}(\cdot, \alpha)$ and $W(\cdot, \alpha)$ are ordinally equivalent.

If the household consumes $x = (w, z) \in E_+^m$ where w is a vector of private goods and z is a vector of pure public goods from the household's point of view,[4] and if the household can make lump-sum transfers to its members, then the household utility function $U(\cdot, \alpha) : E_+^m \longmapsto E$ is given by

$$U(x, \alpha) = \overline{U}(w, z, \alpha) = \max_{(w^1, \ldots, w^l)} \left\{ \widetilde{W}\left(U^1(w^1, z), \ldots, U^l(w^l, z), \alpha \right) \mid \sum_{i=1}^{l} w^i \leq w \right\}, \tag{2.4}$$

where w^i is person i's consumption of private goods and $U^i(w^i, z)$ is his or her utility level, $i = 1, \ldots, l$. $U(\cdot, \alpha)$ measures a utility level that can be assigned to each household member and the vector of equal utilities $(\tilde{u}, \ldots, \tilde{u})$ is ethically indifferent, according to the household's own ethical

3. Interpersonal comparisons of utilities are needed for this.

4. All goods are private goods in the economy. We ignore semi-public household goods for mathematical convenience.

preferences, to the actual utility vector that results from the maximization in (2.4). [5] Household preferences and equally-distributed utility levels are therefore given by a single utility function $U : E_+^m \times \mathbf{A} \longmapsto E$.

If the household social-evaluation function is *maximin*, with

$$\widetilde{W}(u^1, \ldots, u^l, \alpha) = \min\{u^1, \ldots, u^l\}, \tag{2.5}$$

$U(x, \alpha)$ is the utility level of *each* household member. In that case, individuals actually experience the utility level $U(x, \alpha) = \overline{U}(w, z, \alpha)$. This paper is consistent with the more general model, but we write as if the model were rationalized by a maximin social-evaluation function for each household.

Because α can, in principle, name household members, (2.4) is not a serious theoretical restriction. In practice, however, each α describes groups, and (2.4) implies that households with the same characteristics have the same preferences and that, if utility levels are comparable across households, individuals in different households with the same characteristics and consumption vectors are equally well off.

Welfare analyses should be conducted by using a social-evaluation function which depends on the utilities of all n individuals in the economy. In the maximin case, the utility level of (2.5) may be assigned to each person in the household without ethical difficulty. Thus, all people, including children, count in social comparisons. In the general case, however, equally-distributed-equivalent utilities must be used, and this forces investigators to accept the ethics of the household as appropriate for intra-household social decisions.

The indirect utility function V and the expenditure function C corresponding to U are given by

$$u = V(p, y, \alpha) = \max_x \left\{ U(x, \alpha) \mid p \cdot x \leq y \right\} \tag{2.6}$$

and

$$C(u, p, \alpha) = \min_x \left\{ p \cdot x \mid U(x, \alpha) \geq u \right\}. \tag{2.7}$$

For each $\alpha \in \mathbf{A}$, V is continuous and homogeneous of degree zero in p and y, increasing in y, and quasi-convex, nonincreasing and locally nonsatiated in

5. Assume each U^i is continuous, monotonic (nondecreasing and locally nonsatiated) in w and z, and concave, and (given that $W(\cdot, \alpha)$ is continuous, increasing, and quasi-concave) it follows that $U(\cdot, \alpha)$ is continuous, monotonic, and quasi-concave.See Negishi [1963] for a proof in the private-goods case; it can be extended easily to cover pure public goods.

p. C, on the other hand, is continuous in (u,p), increasing in u, and homogeneous of degree one, concave, and nondecreasing in p. In addition C and V are related by the identity

$$y = C(u,p,\alpha) \longleftrightarrow u = V(p,y,\alpha). \tag{2.8}$$

Some household models use a utility function that represents the parents' preferences, which are assumed to include some concern for the well-being of children, if any are present (see Browning [1992]). Although this model may be interpreted in this way, for welfare economics applications I prefer a formulation which permits each individual to count directly. [6]

3. Relative Equivalence Scales

The simplest and probably the most common form of interpersonal comparison is made through the use of adult-equivalence scales. These scales deal with two phenomena. The first concerns the fact that different households contain different numbers and types (adults, children, disabled people, etc.), and therefore have different preferences and needs. The second concerns the fact that there are economies of scale in household consumption (due to public and semi-public consumption within the household). As an example, suppose that a household consists of two adults with an income of \$ 60,000. If we say that the number of adult equivalents in the household is 1.5, then we mean that the household is equivalent, for utility purposes, to two single reference adults with incomes of \$ 40,000 each (\$ 60,000 divided by 1.5). The farther the number of adult equivalents for a two-adult household is below 2, the greater is the economy of scale.

Let d be the number of adult equivalents in a household with characteristics α and income y facing prices p; d is defined implicitly by

$$u = V(p,y,\alpha) = V(p,\frac{y}{d},\alpha^r). \tag{3.1}$$

α^r is the characteristics vector of a reference household with $N(\alpha^r) = 1$ (a single adult).

6. See Blackorby and Donaldson [1991] for other results on equivalence scales which employ this model. In addition, characterizations of the cost-of-children rules proposed by Rothbarth and others are contained in Blackorby and Donaldson [1994].

The definition of d in (3.1) is not meaningful unless the information structure allows comparisons of levels of utilities [7] between members of different households. If V is replaced by $\widetilde{V}$ where

$$\widetilde{V}(p, y, \alpha) = \phi(V(p, y, \alpha), \alpha)) \tag{3.2}$$

and ϕ is increasing in its first argument, then the number of adult equivalents can change. For each α, $\widetilde{V}(\cdot, \cdot, \alpha)$ is ordinally equivalent to $V(\cdot, \cdot, \alpha)$ and each household's preferences are unchanged. If, on the other hand, V is replaced by

$$\widehat{V}(p, y, \alpha) = \psi(V(p, y, \alpha)) \tag{3.3}$$

(ψ increasing), then the number of adult equivalents (defined by (3.1)) remains unchanged. $\widehat{V}$ and V make the same interhousehold comparisons of utility levels.

The mere definition of equivalence scales requires that the information structure support interhousehold comparisons of levels of utility (at least). This is called Ordinal Full Comparability Plus (OFC+), and require that any two utility functions regarded as informationally equivalent be related by (3.3).

(3.1) implicitly defines a function $d = D(u, p, \alpha)$. If the reference adult has an income $y/D(u, p, \alpha)$ and faces prices p, then he or she enjoys a utility level exactly equal to the one enjoyed by each member of a household with characteristics α and income y, facing prices p. Using (2.8), d is given by

$$d = D(u, p, \alpha) = \frac{C(u, p, \alpha)}{C(u, p, \alpha^r)} =: \frac{C(u, p, \alpha)}{C^r(u, p)}. \tag{3.4}$$

It is the expenditure needed to bring each member of a household with characteristics α to utility level u divided by the expenditure needed to bring the reference person to the same level of utility. D is homogeneous of degree zero in p, and $D(u, p, \alpha^r) = 1$ for all u, p.

The function D depends on the utility level of the members of the household, a number that is normally unobservable. A practical solution to this problem is to use a single reference level of utility, u^r, and to define an index

$$\bar{d} = \bar{D}(p, \alpha) = \frac{C(u^r, p, \alpha)}{C(u^r, p, \alpha^r)}. \tag{3.5}$$

7. See Blackorby, Donaldson and Weymark [1984] or Sen [1977] for a discussion.

If u^r is the poverty utility level, then $\bar{d}$ is the ratio of the poverty line for the household in question to the poverty line for the reference household (at the same prices).

$\bar{D}$ uses much less information than D. All that is necessary is that a single indifference surface (corresponding to u^r) be identified for each α. This means that interpersonal comparisons need only be made for a single level of utility. A method for doing this is to find reference consumption bundles $X(\alpha)$, $\alpha \in \mathbf{A}$, such that, for all α

$$U(X(\alpha), \alpha) = u^r. \tag{3.6}$$

If u^r is the poverty utility level, then $X(\alpha)$ is a poverty consumption bundle for a household with characteristics α.

Social evaluations performed with the above indexes of adult equivalence are approximate because the index $\bar{D}(p, \alpha)$ is not always equal to the true adult-equivalence measure $D(u, p, \alpha)$. Social rankings will be correct if and only if the index is exact, that is, if and only if

$$\bar{D}(p, \alpha) = D(u, p, \alpha) \tag{3.7}$$

for all (u, p, α). Exactness therefore requires that D be independent of u. Lemma 1 of Lewbel [1989] demonstrates that the necessary and sufficient condition for this, and therefore for exact social evaluation, is that the expenditure function can be written as

$$C(u, p, \alpha) = \bar{C}(u, p)\widehat{C}(p, \alpha) \tag{3.8}$$

for all (u, p, α). [8]

When (3.8) holds, utilities satisfy *relative equivalence-scale exactness* (RESE). (3.8) is a restriction on U, V, and C; more specifically, it restricts them both *interpersonally* and *intrapersonally*. We can rewrite (3.8) as a relationship between C^r, the reference cost function (derived using (2.7) and (3.4)), the relative exact (independent of u) equivalence scale, and C, the expenditure function. The relative exact equivalence scale D is given by

8. There is a certain amount of arbitrariness here as $\widehat{C}$ can be homogeneous of any degree as long as $\bar{C}$ is homogeneous of one minus that degree. In any case, however, Δ is homogeneous of degree zero in prices. Either $\bar{C}$ or $\widehat{C}$ can be chosen to be homogeneous of degree one; see Blackorby and Donaldson [1991].

$$\Delta(p,\alpha) := D(u,p,\alpha) = \frac{\widehat{C}(p,\alpha)}{\widehat{C}(p,\alpha^r)} \tag{3.9}$$

and in this case (3.7) holds. It is homogeneous of degree zero in p because C is homogeneous of degree one in p, and, of course, $\Delta(p,\alpha^r) = 1$ for all p. Given (3.8) and (3.9), the expenditure function can be rewritten as

$$C(u,p,\alpha) = C^r(u,p)\Delta(p,\alpha). \tag{3.10}$$

This equation shows that one household's preferences may be chosen arbitrarily (corresponding to $C^r(u,p)$). The equivalence scales are allowed to depend on p, a reasonable condition because economies of scale in consumption are likely to be different for different goods and services. Given C^r, the choice of an equivalence scale, Δ, completely determines household preferences for all α, resulting in a significant restriction on preferences. Using (2.8) and (3.10), the indirect utility function V corresponding to C can be written as

$$V(p,y,\alpha) = V^r\left(p, \frac{y}{\Delta(p,\alpha)}\right) \tag{3.11}$$

where $V^r(p,y) := V(p,y,a^r)$.

(3.11) conditions both preferences and interhousehold comparisons of utility. Preferences must be related by

$$V(p,y,\alpha) = \phi\left(V^r\left(p, \frac{y}{\Delta(p,\alpha)}\right), \alpha\right) \tag{3.12}$$

In addition, (3.11) requires $\phi(u,\alpha) = u$ for all (u,a), a condition on interhousehold comparisons.

(3.10) (or (3.11)) admits two special cases: (i) the function $\widehat{C}$ is independent of p, and, therefore, the equivalence scale Δ is independent of p; (ii) $\bar{C}$ is independent of p, and each household's preferences are homothetic. The former is called *Engel equivalence exactness* and the latter *full homotheticity*.

U satisfies Engel equivalence exactness if, for all (u,p,α),

$$C(u,p,\alpha) = C^r(u,p)\Delta^\circ(\alpha) \tag{3.13}$$

and, therefore, for all (p,y,α)

$$V(p,y,\alpha) = V^r\left(p, \frac{y}{\Delta^\circ(\alpha)}\right); \tag{3.14}$$

it satisfies full homotheticity if, for all (u, p, α),

$$C(u,p,\alpha) = \widetilde{C}(u)\check{C}(p,\alpha) \tag{3.15}$$

and, for all (p, y, α),

$$V(p,y,\alpha) = \widetilde{V}\left(\frac{y}{\check{C}(p,\alpha)}\right) \tag{3.16}$$

where $\check{C}$ is homogeneous of degree one in p. Notice that full homotheticity is an interhousehold condition which is stronger than the homotheticity of each household's preferences.

RESE has the consequence that the budget-share equation for any commodity decomposes additively into a function of (u, p) and a function of (p, α). The first of these is the share equation for the reference household, and this means that the income elasticity of demand for any pure children's good such as day-care is *one*. To see this use the theorem of Roy to obtain the demand functions of the household in terms of the demand functions of the reference person :

$$x(p,y,\alpha) = x^r\left(p, \frac{y}{\Delta(p,\alpha)}\right)\Delta(p,\alpha) + y\frac{\nabla_p\Delta(p,\alpha)}{\Delta(p,\alpha)} \tag{3.17}$$

If commodity i is a pure children's good then it is not consumed by the reference adult and hence its demand is given by

$$x_i(p,y,\alpha) = y\frac{\Delta_i(p,\alpha)}{\Delta(p,\alpha)} \tag{3.18}$$

and hence its income elasticity of demand is one.

3.1 Income-Ratio Comparability

Equivalence-scale exactness imposes no restriction on the preferences of a single household, but, given that household's preferences, all the others are linked to it by (3.10). This requirement has implications for interpersonal comparisons of well-being as well.

Suppose that two households face the same prices (with possibly different incomes) and their members enjoy the same level of well-being.

Income-ratio comparability (IRC) is satisfied if the equality of well-being is preserved by arbitrary common scalings of the households' incomes.

Income-Ratio Comparability (IRC): *Utilities satisfy income-ratio comparability if and only if, for all* $p \in E^m_{++}$, $\bar{y}$, $\tilde{y} \in E_+$, $\bar{\alpha}, \tilde{\alpha} \in \mathbf{A}$,

$$V(p,\bar{y},\bar{\alpha}) = V(p,\tilde{y},\tilde{\alpha}) \longleftrightarrow V(p,\lambda\bar{y},\bar{\alpha}) = V(p,\lambda\tilde{y},\tilde{\alpha}) \tag{3.19}$$

for all $\lambda > 0$.

It is tempting to conclude, because income-ratio comparability involves the scaling of incomes, that some kind of interhousehold homotheticity is involved. That is not the case in general (although it is when $U(\cdot,\alpha^r)$ is homothetic) because prices are the same on both sides of (3.19). [9] It is simply a normalization along the income-consumption curves, but these are not required to be straight lines.

Although OFC+ is necessary for income-ratio comparability to be meaningful, IRC imposes structure on the comparisons themselves. Further, it provides a characterization of RESE.

Proposition 1: *Utilities satisfy relative equivalence-scale exactness if and only if they satisfy income-ratio comparability.*

Proposition 1[10] means that, given level comparability of utilities, an axiomatic justification of RESE can be provided by income-ratio comparability.

3.2 Behaviour and Interpersonal Comparisons

In a general model, one can make no inferences about interpersonal comparisons from behaviour alone. Specifically, if the utility function U is replaced with $\widetilde{U}$, where, as in (3.2),

$$\widetilde{U}(x,\alpha) = \phi(U(x,\alpha),\alpha), \tag{3.20}$$

behaviour is the same for all households but interpersonal comparisons between people from households with different characteristics can be changed arbitrarily.

Suppose that there are two different equivalence scales, $\Delta(p,\alpha)$ and $\Delta'(p,\alpha)$, that are consistent with the same household behaviour; then there exist indirect utility functions V^r and $V^{r'}$, and a function ϕ, increasing in its first argument, such that

9. See the discussion of *Comparison Homotheticity* in Blackorby and Donaldson [1987].

10. This is Theorem 4.1 in Blackorby and Donaldson [1993b].

$$V^{r'}\left(p, \frac{y}{\Delta'(p,\alpha)}\right) = \phi\left[V^{r}\left(p, \frac{y}{\Delta(p,\alpha)}\right), \alpha\right] \tag{3.21}$$

for all (p, y, α). Propositions 2 and 3 give necessary and sufficient conditions for (3.21) to hold locally and globally respectively, given a range condition on the ratio $\Delta'(p,\alpha)/\Delta(p,\alpha)$. First it can be shown that this can be true if and only if

$$C(u,p,\alpha) = a(p)[\phi(u)]^{r(p)}\Delta(p,\alpha) \tag{3.22}$$

or, equivalently, that

$$lnC(u,p,\alpha) = r(p)ln\ \phi(u) + ln[a(p)\Delta(p,\alpha)]. \tag{3.23}$$

This requires a local argument only. Using global regularity conditions, it can be shown that monotonicity of the expenditure function in prices and utility requires the function r to be independent of prices, which implies full homotheticity.

As for the needed regularity condition, define the function g by

$$g(p,\alpha) := \frac{\Delta'(p,\alpha)}{\Delta(p,\alpha)}. \tag{3.24}$$

g must be sensitive to α because $g(p,\alpha^r) = 1$ for all p and there exists $\bar{\alpha} \in \mathbf{A}$ such that $g(p,\bar{\alpha}) \neq 1$ (since Δ' and Δ are assumed to be different). Assume, for Propositions 2 and 3, that there exists a $\bar{p} \in E^m_{++}$ such that the range of $g(\bar{p},\cdot)$ contains an interval. This in turn requires some component of α, such as age, to be a continuous variable.

Proposition 2 : *Given income-ratio comparablity (or relative equivalence-scale exactness) , two different equivalence scales, $\Delta'(p,\alpha)$ and $\Delta(p,\alpha)$, satisfying the range condition (above), are consistent with the same locally regular (utility-maximizing) household behaviour, (3.21), if and only (3.22) holds for all (u,p,α), and in that case*

$$\Delta'(p,\alpha) = [S(\alpha)]^{r(p)}\Delta(p,\alpha) \tag{3.25}$$

where $S(\alpha)$ is positive for all α and $S(\alpha^r) = 1$.

Proposition 2 means that (given the range condition) behaviour and the assumption that RESE holds are sufficient to find the equivalence scale uniquely as long as preferences are not log-linear in some transform of utility. Hence the requisite interpersonal comparisons are actually revealed by the the data.

If one is willing to assume that the expenditure function is everywhere increasing in utility and prices, then, a stronger result is available.

Proposition 3 : *Given income-ratio comparablity (or relative equivalence-scale exactness) , two different equivalence scales,* $\Delta'(p,\alpha)$ *and* $\Delta(p,\alpha)$, *satisfying the range condition (above), are consistent with the same globally regular (utility-maximizing) household behaviour, (3.21), if and only if full homotheticity holds for all* (u,p,α), *and in that case*

$$\Delta'(p,\alpha) = S(\alpha)\Delta(p,\alpha) \tag{3.26}$$

where $S(\alpha)$ *is positive for all* α *and* $S(\alpha^r) = 1$.

An objection to the claim that, given RESE and nonhomotheticity, behaviour is sufficient to determine $\Delta(p,\alpha)$ might be that the range assumption in Propositions 2 and 3 is too strong. It is possible, however, to specify sufficient conditions that allow behaviour to determine $\Delta(p,\alpha)$ when $\mathbf{A}$ contains only two (or more) elements. For this, see Theorem 5.3 in Blackorby and Donaldson [1993b].

Given the maintained hypothesis that preferences satisfy RESE/IRC, then behaviour can identify the equivalence scales uniquely. However, this unique identification of the equivalence scales generates, in addition, the interpersonal comparisons of utility.

4. Absolute Equivalence Scales[11]

In this section I repeat briefly all the arguments presented in the previous section with absolute equivalence scales. This small change leads as will be seen to rather surprising results.

Let a the absolute adult equivalence scale for a household with characteristics α and income y facing prices p; it is defined implicitly by

$$u = V(p,y,\alpha) = V(p,y-a,\alpha^r). \tag{4.1}$$

This defines implicitly a function $a = A(u,p,\alpha)$. It is retrieved using (2.8) and is given by

$$A(u,p,\alpha) = C(u,p,\alpha) - C(u,p,\alpha^r). \tag{4.2}$$

11. This material is taken from Blackorby and Donaldson [1994].

This equivalence scale is exact (Absolute Equivalence Scale Exactness — AESE) if and only if the expenditure function can be written as

$$C(u,p,\alpha) = \widehat{C}(u,p) + \bar{C}(p,\alpha) \tag{4.3}$$

in which case the equivalence scale can be written as

$$A(p,\alpha) = \bar{C}(p,\alpha) - \bar{C}(p,\alpha^r). \tag{4.4}$$

Note that the equivalence scale of the reference household is now zero and not one as before. Using this definition the expenditure function of each household can be written as

$$C(u,p,\alpha) = C^r(u,p) + A(p,\alpha) \tag{4.5}$$

the indirect utility function can be written as

$$V(p,y,\alpha) = V^r(p,y - A(p,\alpha)) \tag{4.6}$$

Using the theorem of Roy, the demands of a household with income y, characteristics, α, and facing prices, p, are given by

$$x(p,y,\alpha) = x^r(p,y - A(p,\alpha)) + \nabla_p A(p,\alpha). \tag{4.7}$$

Thus, if i is a pure children's good, its demand is given by

$$x_i(p,y,\alpha) = A_i(p,\alpha). \tag{4.8}$$

Thus, the income elasticity of demand for a pure children's good is zero. This is in marked contrast with the relative equivalence scale where the income elasticity of a pure children's good is one.

4.1 Income-Difference Comparability

Absolute equivalence-scale exactness imposes no restriction on the preferences of a single household, but, given that household's preferences, all the others are linked to it by (4.6). This requirement has implications for interpersonal comparisons of well-being as well.

Suppose that two households face the same prices (with possibly different incomes) and their members enjoy the same level of well-being. *Income-difference comparability* (IDC) is satisfied if the equality of well-being is preserved by arbitrary common additions or subtractions of the households' incomes.

Income-Difference Comparability (IDC): *Utilities satisfy income-difference comparability if and only if, for all* $p \in E_{++}^{m}$, $\bar{y}$, $\tilde{y} \in E_{+}$, $\bar{\alpha}, \tilde{\alpha} \in \mathbf{A}$,

$$V(p,\bar{y},\bar{\alpha}) = V(p,\tilde{y},\tilde{\alpha}) \longleftrightarrow V(p,\bar{y}+\delta,\bar{\alpha}) = V(p,\tilde{y}+\delta,\tilde{\alpha}) \tag{4.9}$$

for all δ *such that both incomes remain positive.*

IDC is a form of interpersonal comparisons in addition to that imposed by ordinal level comparability. In fact, it is an alternative characterization of absolute equivalence scale exactness.

Proposition 4: *Utilities satisfy absolute equivalence-scale exactness if and only if they satisfy income-difference comparability*

4.2 Behaviour and Interpersonal Comparisons

An argument about recovering equivalence scales and the attendant interpersonal comparisons from behaviour is also available for absolute scales provided an analogue of the above range conditions is met.[12] This is stated as

Proposition 5: *Given IDC (or equivalently AESE), two different absolute equivalence scales,* $A'(p,\alpha)$ *and* $A(p,\alpha)$*, satisfying the range condition, are consistent locally with the same (utility maximizing) behaviour if and only if*

$$C(u,p,\alpha) = \widehat{C}(p)\phi(u) + \bar{C}(p) \tag{4.10}$$

where

$$A'(p,\alpha) = \widehat{C}(p)S(\alpha) + A(p,\alpha) \tag{4.11}$$

and $S(\alpha^{r}) = 0$.

Without repeating the above argument, global recovery requires full homotheticity.

5. Remarks

From the above two sections one sees that simple changes in assumptions can lead to rather important changes in the resulting application. In this case, the choice between relative equivalence scale exactness and absolute equivalence scale exactness. How would one choose between the two. In this case, it seems easy, that is, it is unlikely that the income elasticity of demand for pure children's goods is zero. But it is important to

12. This requires a range condition of the difference $A'(p,\alpha) - A(p,\alpha)$.

be careful here, what assumptions can one actually test? The list of assumptions employed here is long : that the family is maximizing some social welfare function that can be represented as above, that level comparisons of utility can be made, as well as some form of equivalence scale exactness.

Empirically one can test whether or not the data could have been reasonably generated by some functional form that satisfies the requisite exactness condition. One problem here is that the functional forms often employed frequently cannot actually test the exactness hypothesis. A theoretical discussion of the AIDS and Translog systems is in Blackorby and Donaldson [1993b] and their empirical tests in Blundell and Lewbel [1991] and Dickens, Fry and Pashardes [1993]. More recently a semiparametric test has been conducted by Pendakur [1998]. There are however many alternative equivalence scale assumptions that have been proposed ; among them are the Rothbarth method, the iso-proportional method, and the assumption of demographic separability.[13] For the most part, none of these alternatives is nested in the other.

Recently, Donaldson and Penakur [1999] have proposed a method that is more general than those discussed above which seems promising on both the theoretical and empirical grounds. They generalize both RESE and AESE so that the restrictions on pure children's goods are no longer constants. In fact their generalized absolute equivalence scale exactness condition permits the demand for a pure children's commodity to be affine in income. In addition they demonstrate that these scales can under certain circumstances be identified uniquely from the data.

References

BLACKORBY C., DONALDSON D., Welfare ratios and distributionally sensitive cost-benefit analysis. *Journal of Public Economics*, 1987, 34, 265–290.

BLACKORBY C, DONALDSON D, Money metric utility : a harmless normalization ? *Journal of Public Economics*, 1988, 46, 120–129.

13. For references and the conditions that are necessary and sufficient for the exactness of these, see Blackorby and Donaldson [1994].

BLACKORBY C., DONALDSON D., Adult-equivalence scales, interpersonal comparisons of well-being, and applied welfare economics. in Elster J, Roemer J, eds., *Interpersonal Comparisons and Distributive Justice.*, 1991, Cambridge University Press, Cambridge, 164–199.

BLACKORBY C., DONALDSON D., Household equivalence scales and welfare comparisons : a comment. *Journal of Public Economics*, 1993a, 50, 143–146.

BLACKORBY C., DONALDSON D., Adult-equivalence scales and the economic implementation of interpersonal comparisons of well-being, *Social Choice and Welfare*, 1993b, 10, 335-361.

BLACKORBY C., DONALDSON D., Measuring the cost of children : a theoretical framework. Blundell R., ed., *The Measurement of Household Welfare.*, 1994, Cambridge University Press, Cambrdidge, 51-69.

BLACKORBY C., DONALDSON D., Weymark J, Social choice with interpersonal utility comparisons : a diagrammatic introduction. *International Economic Review*, 1984, 25, 327–356.

BLUNDELL R., LEWBEL A., The information content of equivalence scales. *Journal of Econometrics*, 1991, 50, 49–68.

BROWNING M., Children and household economic behaviour. *Journal of Economic Literature*, 1992, 30, 1434–1475.

DICKENS R., FRY V., PASHARDES P., Non-linearities, aggregation and equivalence scales. *Economic Journal*, 1993, 103,359-368.

DONALDSON D., PENDAKUR K., Equivalent-income function and income-dependent equivalence scales. Department of Economics Discussion Paper, 99-16, *University of Britsh Columbia*, 1999.

LEWBEL A., Household equivalence scales and welfare comparisons. *Journal of Public Economics*, 1989, 39, 377–391.

LEWBEL A., Cost of characteristics indices and household equivalence scales. *European Economic Review*, 1991, 35, 1277–1293.

NEGISHI T., On social welfare function. Quarterly *Journal of Economics*, 1963, 77, 156–158.

SAMULESON P., Social indifference curves. Quarterly *Journal of Economics*, 1956, 70, 1–22.

SEN A., On weights and measures : informational constraints in social welfare analysis. *Econometrica*, 1977, 45, 1539–1572.

WEYMARK J., Money-metric utility functions. *International Economic Review*, 1985; 26, 219–232.

Résumé

Les échelles d'équivalence adulte sont une tentative d'employer les données de marché pour comparer les niveaux d'utilités entre des membres des ménages de compositions et de tailles différentes. Par exemple, ce qui est le seuil de pauvreté pour une famille de deux adultes et deux enfants par rapport à une famille d'un adulte et trois enfants. L'article présente un petit tour d'horizon des échelles d'équivalence. On procède en examinant deux échelles différentes. Les hypothèses requises pour établir les échelles d'équivalence qui sont relatives et exactes ont été présentées dans un certain détail. Etant donné qu'on peut faire des comparaisons des niveaux d'utilité, les circonstances sous lesquelles ces comparaisons pourraient être révélées par des données de marché sont établies. Alors, faisant un changement dans l'ensemble des hypothèses, les échelles d'équivalence exactes et absolues sont présentées dans un mode parallèle. Certaines remarques méthodologiques concluent.

Abstract

Adult equivalence scales are basically an attempt to use market data to make interpersonal comparisons of levels of utility between members of households of different sizes and compositions. For example, what is the poverty line for a family of two adults and two children as opposed to a family of one adult and three children. The paper presents a brief survey of equivalence scales and proceeds by examining two different types of equivalence scales. The assumptions needed to establish relative exact equivalence scales are set out in some detail. Given that one can make comparisons of levels of utility, the circumstances under which these comparisons could be revealed by market data are established. Then, making one change in the set of assumptions, absolute exact equivalence scales are presented in a parallel fashion. Some methodological remarks conclude.

Mots-clé

Échelles d'équivalence, données de marché, niveaux d'utilité

Keywords

Equivalent scales, market data, levels of utility

JEL

031, D6, H2.